Exp. 3 USE OF BASIC ELECTRONIC MEASURING INSTRUMENTS Part II, & ANALYSIS OF THE MEASUREMENT ERROR

PURPOSE:

- To become familiar with more of the instruments in the laboratory.
- To become aware of operating limitations of input and output devices.
- To understand and be able to work with root mean square (RMS) measurements.
- To learn the principles of analyzing the measurement error, including variance and Gaussian distribution.

This experiment relates to the following **learning objectives** of the course

1. Ability to interconnect equipment and devices such as multimeter, function generator, and oscilloscope to achieve required results.
2. Acquire practice in recording data and results and maintaining a proper engineering notebook.
3. Ability to analyze and evaluate data.

LAB EQUIPMENT:

1 Agilent E3640A DC Power Supply
1 Agilent 34410A Digital Multimeter/Timer/Counter
1 Agilent 33120A Function Generator (FG)
1 Agilent 54621A Oscilloscope

STUDENT PROVIDED EQUIPMENT:

1 BNC-Double Banana
1 BNC T Adapter
1 BNC-BNC Cable

Experiment Sections:

1) Agilent 33120A Function Generator and Agilent 54621A Oscilloscope
2) Voltage Measurements at Various Frequencies
3) Function Generator/Counter Accuracy
4) Period Measurement with the Frequency Counter
5) Analysis of the Measurement Error

Section 1) Agilent 33120A Function Generator Function Generator (FG)

Background

Note:

- In the section 1 of this experiment, you will configure the FG to supply seven different waveforms. Each of these waveforms will be initially stored by the FG, which will allow you to recover them quickly. There are some instructions on how to operate the FG given below. Please use the manual for reference.

- You must first change the Output Termination Impedance of the FG to "High Z" by following the steps outlined in page 40 of the FG operating manual.

- You can store up to 3 waveforms at a time with the Agilent 33120A Function Generator.

- The maximum DC Offset that can be generated by the Agilent 33120A Function Generator is given in the operating manual as:

$$\left|V_{Offset}\right| < V_{\max} - \frac{V_{pp}}{2} \quad \text{and} \quad \left|V_{Offset}\right| < 2V_{pp} \text{ whichever is less}$$

Where V_{max} = 10V for a High-Impedance termination and 5V for a 50Ω termination.

RMS Value of a Waveform

Sinusoidal signals, e.g., alternating current or voltage waveforms (AC signals), are periodic and hence change with time. For these signals, the *instantaneous power*, which is proportional to the square of the signal at any given point of time, is also periodic and hence changes with time. However, for these signals, we are often interested in the *average power* and not care for the instantaneous power. The root-mean-square (RMS) value (defined below) is a *single* measure (value) of a sinusoidal signal that can be used in the average power calculations. The RMS value is also called the *effective value* because it is *effectively* like a DC value, i.e., it can be used in the power formula like a DC value. For example, assume that we apply a DC voltage $\mathbf{V_{DC}}$ to a resistor **R** causing a DC current $\mathbf{I_{DC}}$ through it. The DC power, which of course is a constant and is the same as the average power, is:

$$\mathbf{Power_{DC\ source} = V_{DC}\, I_{DC} = V^2_{DC} / R = I^2_{DC} R}$$

For AC source, we use the same formula except we use the RMS value of voltage and current. That is,

$$\mathbf{Power_{AC\ source} = V_{RMS}\, I_{RMS} = V^2_{RMS} / R = I^2_{RMS} R}$$

The RMS value of a function is defined as the square root of the average value (*mean*) of the square of a function. If a function ***v(t)*** is periodic with a period of ***T***, then the RMS of this function is mathematically defined as:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

To obtain the RMS value of a function, we follow these three steps:

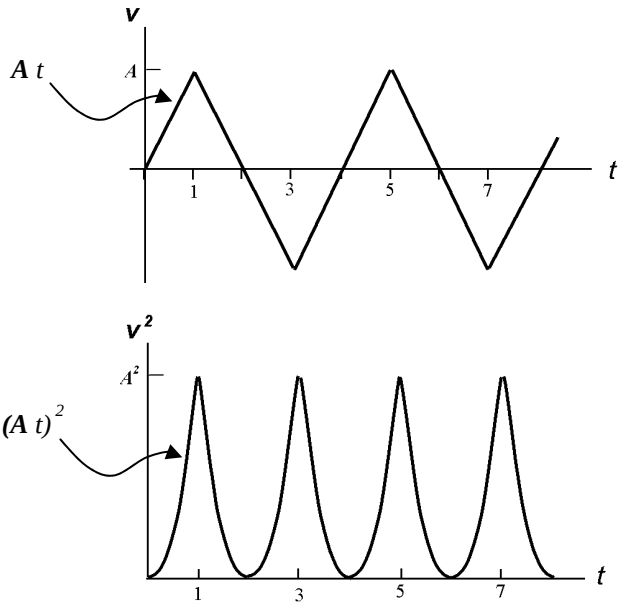
1. Square the function over a period
2. Calculate the mean of this new function
3. Take the square root

Example

Assume we have a triangular function with a period of $T = 4$ and an amplitude of A with no DC offset. Because of symmetry, it suffices to integrate v^2, from 0 to 1, i.e., to average over a quarter period, *to* find the RMS value:

$$V_{RMS} = \sqrt{\frac{1}{1}\int_0^1 (A\,t)^2\,dt}$$

$$= \sqrt{\frac{A^2}{3}\,[t^3]\Big|_{t=0}^{t=1}} \qquad = \frac{A}{\sqrt{3}}$$



The RMS of sine waveforms can be obtained similarly as done in the pre-lab.

Experimental procedure:

Note: throughout this experiment, set the FG to "High Z" output termination Impedance

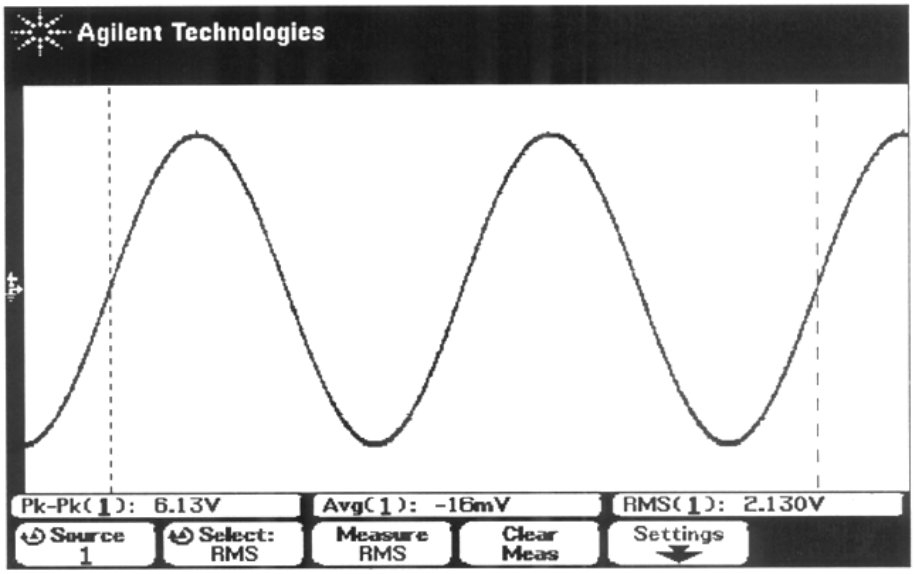
a) Configure and store the following waveforms in the FG. You should be able to figure out how to store waveforms (up to a maximum of three) in the Agilent 54621A oscilloscope, otherwise you can find out how by checking its user's manual (also available on line).

1) 6.00 V_{pp} sine wave @ 500 Hz with no DC offset
2) 2.12 V_{rms} sine wave @ 500Hz with no DC offset
3) 2.00 V_{rms} sine wave @ 1k Hz with 2.0 V DC offset

Note: for each waveform, enter its parameters into the function generator exactly as stated. In particular, this will require you to switch back-and-forth between V peak-to-peak and V RMS (by pressing Enter Number, then either ∧ or ∧).

b) Recall each of the three stored waveforms one at a time and perform the following measurements:
1) AC (Peak-to-Peak)
2) DC (Average)
3) RMS values

Perform these measurements on the oscilloscope only. To do so, press Quick Measure button, then Select and Measure each of the following: AC (Peak-to-Peak), DC (Average), and RMS. For example, here is the results that were obtained for the first waveform:



(Note how, as indicated by the dashed lines, the scope automatically performs the calculations over an integer number of periods of the periodic waveform.) Always be sure the vertical sensitivity of the scope is set so the waveform is not clipped; otherwise, it might perform the calculations incorrectly.

c) Repeat steps a and b for the following three waveforms

4) 2.00 V_{rms} sine wave @ 1k Hz with –2.0 V DC offset
5) 5.00 V_{pp} triangle wave @ 2K Hz with no DC offset
6) 6.00 V_{pp} square wave @ 2K Hz with no DC offset

d) Repeat steps, a and b for this waveform

7) 6.00 V_{pp} square wave @ 2K Hz with 1.0V DC offset

e) Display the results (AC, DC, and RMS values) in tabular form for all seven waveforms.

Questions: Section 1

1) How closely do waveforms 1 and 2 match each other for the AC reading? Why is that?

2) What would you mathematically expect waveform numbers 3, 4, and 5 to be for RMS values? How close are these predictions to the actual scope readings?

3) How closely do the measurements for waveforms 6 and 7 match what was calculated in question 3 and 4 of the prelab?

Section 2) Voltage Measurements at Various Frequencies

a) Configure the FG to supply a 3.0 V_{rms} sine wave @ 1 Hz with no DC offset.

b) Using the AC scale on the Agilent 34401A multimeter, measure the voltage output of the FG. You should observe that at this very low frequency, the multimeter display jumps around!

c) Repeat steps a) and b) for the following frequencies: 10 Hz, 100 Hz, 1 kHz , … , 10 M Hz. Calculate the % difference from the original 3.0 V_{rms}. Put the results in tabular form.

Question: Section 2

What trend is evident in the percent error?

Section 3) Function Generator/Counter Accuracy

a) Configure the FG to supply a 3.0 V_{rms} sine wave @ 500 Hz with 0V DC offset.

b) Use the Agilent 34401A Counter to measure this frequency and record the results.

c) Repeat steps a) and b) for the following frequencies: 10 Hz, 100 Hz, 1 kHz , … , 10 M Hz. Calculate the % difference from the original 3.0 V_{rms}. Put the results in tabular form.

Questions: Section 3

What trend is evident in the percent error?

Section 4) Period Measurement with the Frequency Counter

a) Configure the FG to supply a 3.0 V_{rms} sine wave @ 1 Hz with no DC offset. Try to measure the period of this waveform using the counter. Again, as in section 2 (a), you should observe that the multimeter display jumps around at this very low frequency (<2 Hz).

b) Configure the FG to supply a 3.0 V_{rms} sinewave @ 100 Hz.

c) Measure and record the period of this waveform using the counter.

d) Repeat steps a) and b) for frequencies of 10 Hz, 100 Hz, 1 kHz , … , 10 M Hz. Calculate the % difference from the original 3.0 V_{rms}. Put the results in tabular form.

Questions: Section 4

What trend is evident in the percent error?

Section 5) Analysis of the Measurement Error

In this section, we learn to analyze the measurement error.

Overview of Error Analysis

Data collection and interpretation is key to successful laboratory experimentation. There are two main types of experimental errors in physical measurements.

> ***Systematic Errors*** will cause the entire distribution of data points to be offset with respect to the true value. Causes of systematic error include poor measurement technique, errors in instrumental calibration, software errors or failure to correct for external conditions (e.g. temperature). Possible sources of systematic error must be considered in all of the stages of the experiment, from design to data analysis.
>
> ***Random Error*** is the deviation from the true value which occurs in any physical measurement. The assumption is that this error will result in experimental readings which are equally distributed between "too high" and "too low," and the mean value reflects the true average value, to within some precision.

Analysis of Random Error

Suppose you want to measure a physical property, e.g., the RMS value of a waveform. The numerical values that the physical quantity can assume may be represented by a *random variable* ***R*** whose *population* is the set $\{X_1, X_2, X_3, \ldots, X_n\}$. Each X_i represents a possible value or *outcome*. Note that the population of ***R*** may include multiple occurrences of the same value. Two important statistical properties of this set are the average value (*mean*) and the standard deviation (*standard error*) defined as:

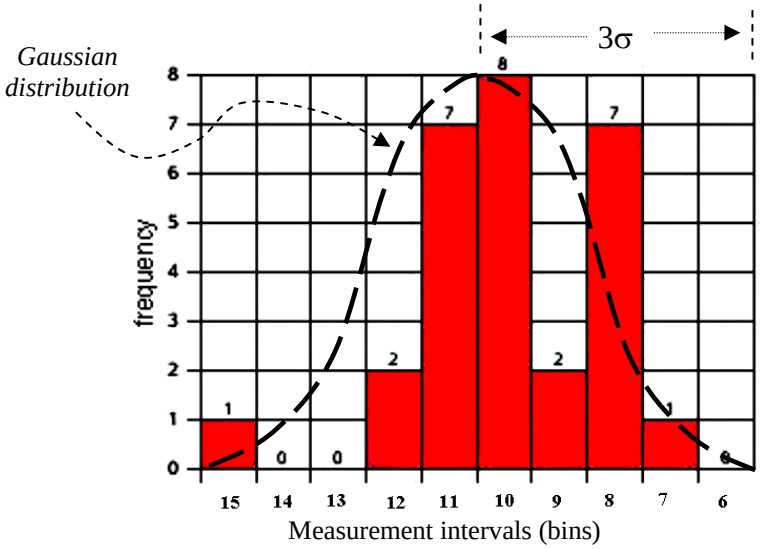
Average *(mean):* $$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Standard deviation *(standard error):* $$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}$$

The Standard Deviation (also known as Standard Error) measures the spread of the data about the mean value. It represents the expected error in a single measurement of a physical quantity, i.e., the expected deviation from its true mean. In laboratory measurements, we can only obtain a subset of the entire population (i.e., all possible outcomes of a measurement) of ***R***. For this reason, we can only obtain estimates of the true average and standard error. If a measurement is carried out m times, where m < n, then we can use the above equations to estimate the average and standard error by replacing n with m. This means that more measurements translate to smaller errors in the estimate of the true mean and standard error. Note that the measurements are assumed to be unbiased and carried out under identical conditions.

Gaussian distribution

The standard error does not provide a complete representation of the measurement error spread (deviation from the mean). To obtain a complete distribution of the error, we can divide the range of values between the minimum and maximum into m equal intervals (or bins) and plot the frequency of occurrence for each range of values within each bin. For many physical quantities, the distribution has a bell-shaped form known as a *Gaussian* or *normal* distribution. For a Gaussian distribution, 68.3 percent of the measurements are within one standard deviation of the mean. As shown, almost all measured values fall within $\pm 3\sigma$ of the mean. The distribution has the mathematical form:



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\overline{X})^2}{2\sigma^2}\right)$$

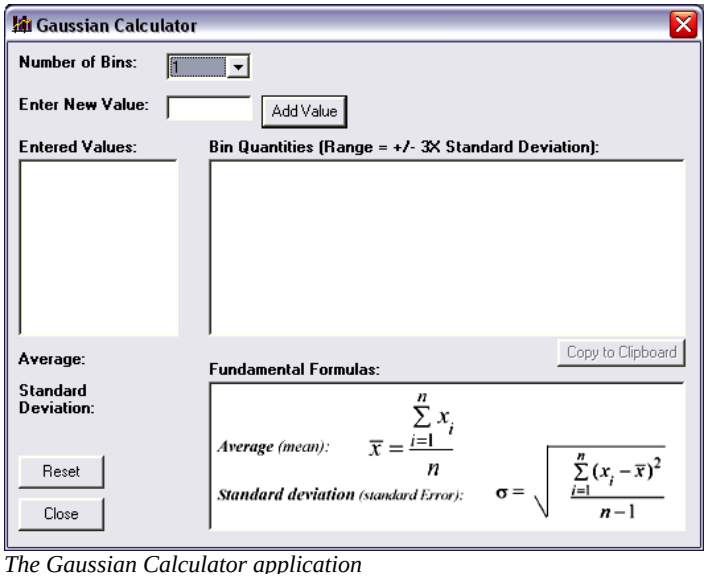
Experimental Procedure

Note:

An executable program "The Gaussian Calculator" has been designed in order to assist in the construction of Gaussian distribution plots required in parts (e) through (*g*). This is done by categorizing entered values into specific bins of a predetermined size. Once all the values have been entered, the data is exportable to any spreadsheet applications. The Gaussian Calculator is also useful in quickly calculating the average value and standard deviation. The executable code and the user guide are available either from the EE 241 directory or by clicking the following links:

Gaussian Calculator User Guide:

Gaussian Calculator Executable:



The Gaussian Calculator application

a) Configure and store the following waveforms in the Agilent function generator

V_{pp} sine wave @ 1 KHz with no DC offset

b) Connect the FG output to the Agilent oscilloscope (for this experiment, you can use a black coaxial BNC to BNC cable)

c) Set the scope to AC coupled and press the "Quick/Meas" button on the scope panel and then select RMS measurement. You will notice that the digital readout (for the RMS value of the waveform) of the scope is not stable; the least significant digit is constantly changing. The continually changing readout can be assumed to represent the varying outcomes (values) of a measurement process (RMS value of the FG output waveform).

d) Freeze the waveform by pressing the "Stop/Run" button on the scope panel (thereby making it turn red) to read and record the digital readout at any moment. Once the Sop/Run button of the scope has been pressed, successive individual traces can be obtained by pressing the Single button. Obtain 10 RMS readings and record them in a table.

e) Calculate the mean and the standard deviation for the set of ten measurements.

f) Increase the number of readings to 20 and recalculate the mean and standard deviation. Record the differences from the previous values in your table.

g) Increase the number of measurements to 40 and plot the distribution of the error (deviation from the mean).

h) Manually fit the above standard error distribution to a Gaussian distribution function as shown in the figure above. Estimate the standard deviation from the manually fitted Gaussian curve (Remember the curve spans $\pm 3\sigma$ about the mean) and record the percent difference from the calculated value.

Questions: section 5

1) How and why do the estimates of the mean and standard deviation change as we increased the number of samples from 10 to 20?

2) How close is the mean value (based on the 20-point sample set) to the theoretical value? If we increase the number of measurements to say 1000, will we nearly reach the theoretical value?

3) For error analysis, would it be acceptable if you obtain additional measurement values from students on other benches?

4) What change do you expect in the difference between the calculated and manually measured values of the standard error as the number of measurements increase? (explain).

5) Do you expect the error distribution look more "normal" as the number of measurements increase?